# Certain Factors Affecting the Gain of Directive Antennas*

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This paper analyzes the performance of antenna arrays as influenced by certain variables within the control of the designing engineer. It starts with an extremely simple analysis of the interfering effects produced by two sources of waves of the same amplitude. This is followed by a short discussion of a paper by Ronald Foster, which considers two antennas and also 16 antennas when arranged in linear array. Two antennas separated in space by ¼ wave-length and in phase by ¼ period give sensibly more radiation in one direction than in the opposite. This, for convenience, has been called a unidirectional couplet. A number of these couplets may be arranged in linear array, thereby giving an extremely useful directive system. Diagrams are shown for such arrays as affected by the number and spacings of the individual couplets. The gains from such arrays are calculated and data are given showing fair agreement between calculation and observation.

Directional diagrams for arrays of coaxial antennas indicate that somewhat less gain may be expected from this form than when the elements are spaced laterally. Combinations of these two types of arrays give marked directional properties in both their horizontal and vertical planes of reference. This principle has been used rather generally in short-wave communication. This paper also discusses effects resulting from combining two or more arrays. In one case the space between two arrays tends to emphasize spurious lobes. The directional diagram of such a combination may be rotated within limits by changing the phasing between adjacent arrays or sections of an array. In all of the above cases the influence of the earth is ignored.

A mathematical appendix gives general equations for calculating directional diagrams of linear arrays. Special cases of these equations apply to the figures included in the main part of the text. General equations are also given for calculating the gains of arrays. Similar equations permit the areas of diagrams to be calculated. An extended bibliography on antenna arrays is appended.

## Introduction

THROUGHOUT the development of radio communication the engineer has aspired to a directive system whereby radiation might be projected from one point to another with a maximum of efficiency and a minimum of interference with adjacent stations. Also, he has aimed at similar directivity at the receiver to improve the signal-to-noise ratio and otherwise discriminate against undesirable signals. It was recognized at a very early date that directive radio based on wave interference was feasible provided sufficiently short waves could be utilized, and as a result many interesting suggestions to this end were made. However, as is well known, the early development of the radio spectrum proceeded in the direction of long

* Presented at Convention of I. R. E., Toronto, Ont., Canada, Aug. 19, 1930. *Proc., I. R. E.*, Sept. 1930.

waves rather than short waves, thereby deferring many of the applications of these suggestions.

The principle of wave interference on which most short-wave systems of directive radio are based has probably been known for several centuries. However, the first thorough treatment of this subject was by Sir Thomas Young,[1] who, together with Fresnel, securely established the wave theory of light in the early part of the last century. Even Hooke and Huygens, who had offered the wave theory over a century earlier, failed to recognize the full significance of interference.

When Hertz started his celebrated experiments to verify Maxwell's theory he was, of course, in full knowledge of these phenomena and their explanation, and invoked their use in proving the existence of electric waves. It is interesting that in some of his experiments he made use of parabolic mirrors for both transmitting and receiving, having directional characteristics very similar to those sometimes used in present day radio practice. It is also of interest that he found that parallel wires stretched over a frame were quite as effective as a reflector as a continuous sheet of metal of similar dimensions, provided the wires were kept parallel to the lines of electric force of the arriving wave. He apparently did not investigate the effect of varying the spacing nor the length of the parallel wires, nor did his subsequent experiments otherwise tend toward the present day antenna array technique.

This paper treats in an elementary way certain aspects of the antenna array problem, principally as regards the manner in which calculated directivity is affected by the number and spacing of the individual antennas which go to make up the array. The theory is applicable only to those forms of directive antennas which may be resolved into a series of individual sources. It does not apply to the so-called wave antenna. However, principles are included which have for some time been in general use in combining two or more such antennas.

Extensive study has been given to directive antenna systems for use in transoceanic radiotelephony. Papers dealing with this general subject have appeared from time to time.[2] Further work is in progress. Papers by E. J. Sterba and also by E. E. Bruce and H. T. Friis of the Bell Telephone Laboratories are in preparation which will include

[1] *Phil. Trans. of Royal Soc.*, **92**, 12; 1802.

[2] R. M. Foster, "Directive diagrams of antenna arrays," *Bell Sys. Tech. Jour.*, 292, May, 1926. Austin Bailey, S. W. Dean, and W. T. Wintringham, "Receiving system for long-wave transatlantic radiotelephony, *Proc. I. R. E.*, **16**, 1694, December, 1928. J. C. Schelleng, "Some problems in short-wave radiotelephone transmission," *Proc. I. R. E.*, **18**, 913; June, 1930.

certain calculated data similar to those contained in the present paper, and also experimental results obtained from tests on actual antennas of various sizes and proportions.

In the early part of the following discussion each antenna is considered as a spherical source of waves which radiates equal power in all directions. Furthermore, it assumes that the current in each

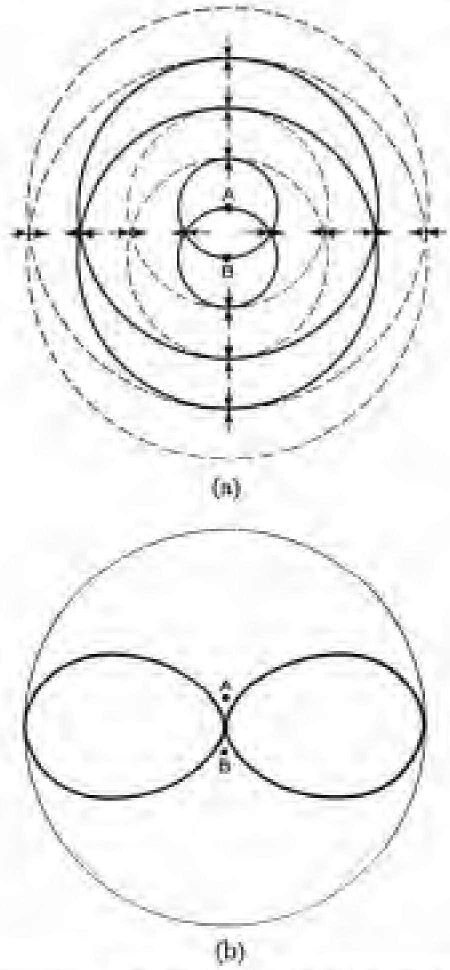



Fig. 1—Interference pattern. Two equiphased sources spaced one-half wave-length.

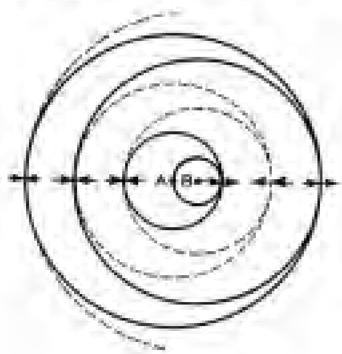
individual source, in a given array, is the same and is not materially affected in either magnitude or phase by its proximity to other sources. The fair approximation to which these calculated results are realized in practice bespeaks the justification of these assumptions.

The various steps by which present day directional radio has been developed are extremely interesting, but they are so involved in the development of radio itself that their enumeration is considered out-

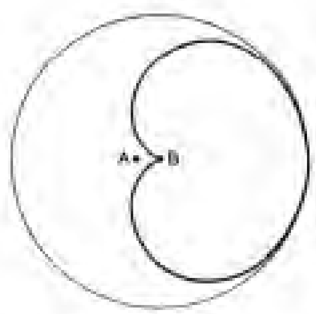
side the scope of this paper. However, bibliographies are cited below covering some of their important phases.

## Elementary Principles

The interference patterns resulting from a number of individual sources of waves, such as antennas, are dependent on both their spacial arrangement and the magnitudes and relative phases of their forces. This makes possible an almost unlimited number of combinations of which only a portion have thus far found use in communication. This paper will restrict itself mainly to some cases which are already finding general application. As a suitable introduction to this subject, a very simple case of wave interference is discussed in the following paragraph.




(a)




(b)

Fig. 2—Interference pattern. Two sources separated in space by one-fourth wave-length and in time by one-fourth period.

Figs. 1a and 2a depict in a rough way the interference resulting from two independent sources of spherical waves of the same amplitude. In the first case they are spaced ½ wave-length but are assumed to be oscillating in phase. In the second case the two sources are separated in space by ¼ wave-length and in phase by ¼ period. Crests

and troughs are represented respectively by solid and dotted lines. At points where either two crests or two troughs arrive simultaneously the resultant wave is greatly enhanced, whereas at certain other points crests and troughs arrive together, thereby neutralizing each other's effects. At certain intermediate points these interfering effects are only partially complete. Accompanying each figure is a directive diagram (1b and 2b), plotted in polar coordinates, which shows the effectiveness of the wave in each direction. The circle drawn outside each diagram indicates the effect if the radiation had proceeded from a single non-directional source similar to each of the above. The ratio between the areas of the circle and the inscribed diagram gives roughly the power improvement of such a device as manifested in the intensity of the radiated wave. A more exact calculation of this improvement requires an integration of the force components over a unit sphere.

## Linear Antenna Arrays

Most directive antenna systems now in general use for short waves may be regarded as special applications of the linear array. This type consists of two or more antennas all having currents of equal amplitude, equispaced along the same straight line. The properties of such arrays have been treated very generally by Foster,[2] whose paper included several hundred directive diagrams, taken in a bisecting plane perpendicular to the axis of each antenna of the array, and typical of the results which may be expected from two antennas and from arrays consisting of 16 antennas. A portion of these diagrams have been reproduced in Figs. 3 and 4 below. The same principles are applicable to both transmission and reception.

In Fig. 3 are shown diagrams resulting from two antennas as the separation is increased from 0 to 1 wave-length in steps of $\frac{1}{8}$ wave-length and the phase increased from 0 to $\frac{1}{2}$ period in steps of $\frac{1}{8}$ period. The line or axis of the array is assumed to be horizontal and the specified phase difference is such that the current in the right-hand antenna is lagging for a transmitting system and leading for a receiver. It will be noted that for phase differences of both 0 and $\frac{1}{2}T$ the diagrams are symmetrical about both the horizontal and vertical axes of the figure, whereas for other phases the figures are asymmetrical about the vertical axis except for certain limiting cases. Of these asymmetrical diagrams, that corresponding to phase and spacial separations both of $\frac{1}{4}$ (Fig. 3b) is of particular importance and forms the basis of the so-called reflector effect. This particular combination of two sources is referred to later as a unidirectional couplet.[3] In

[2] Loc. cit.

[3] In this, and in other cases in this paper, radiation is referred to as unidirectional when sensibly more power is propagated in one direction than in others.

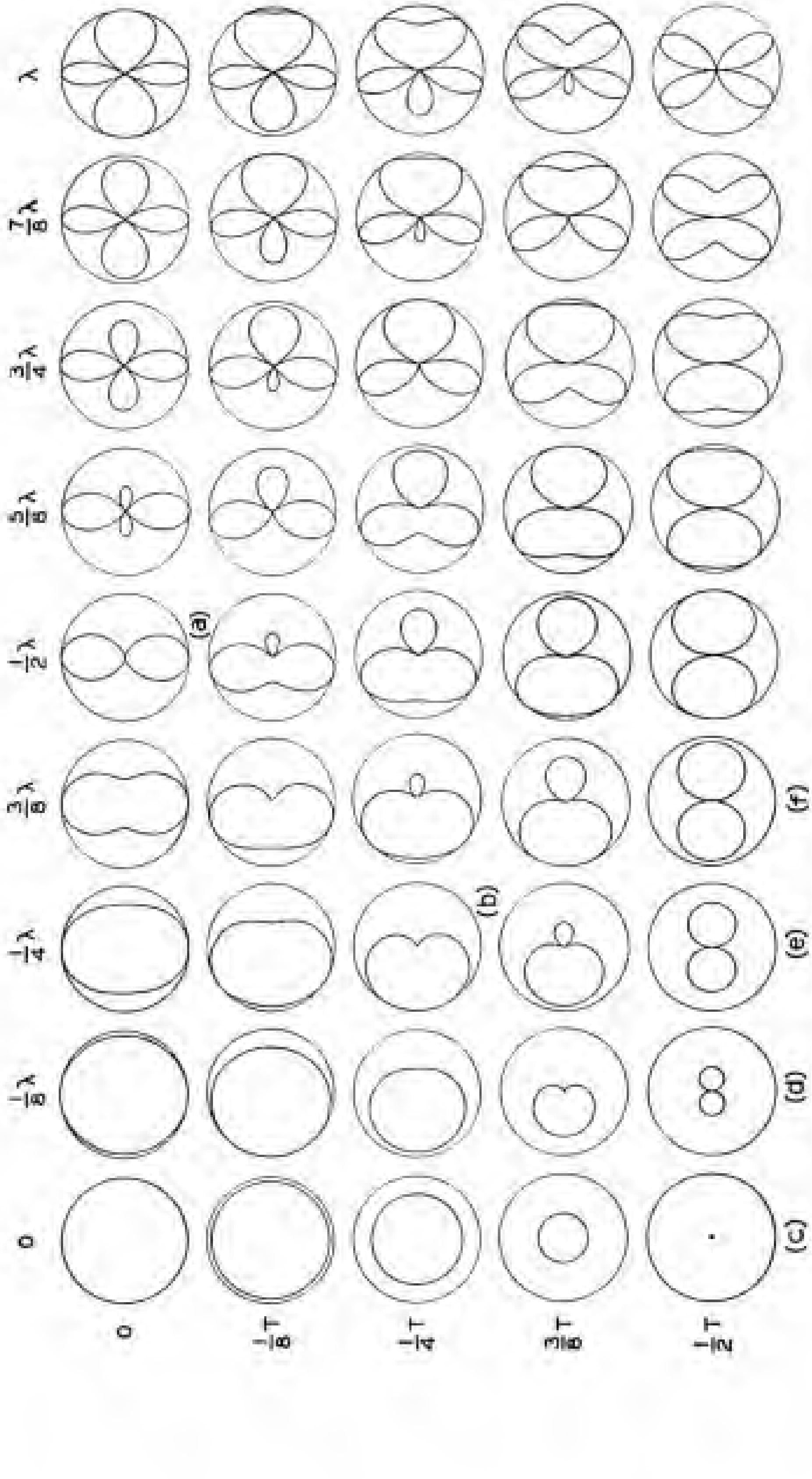



Fig. 3—Directive amplitude diagrams for an array of two antennas. Separation in wave-lengths ($\lambda$) along the top. Phase difference in periods (T) at the left.

passing it is also of interest to note that the diagram of the coil or frame aerial as generally used is intermediate between Figs. 3c and 3d. Its diagram would not differ essentially from its neighbors, Figs. 3d, 3e, or 3f, except for scale. This scale may conveniently be regarded as a measure of the impedance of the device, or possibly its radiation efficiency, but not necessarily a measure of its usefulness.

Fig. 4 shows similar diagrams resulting from 16 antennas for various phase and space relations. As in Fig. 3, diagrams in the top and bottom rows corresponding respectively to phases of 0 and ½ $T$ are symmetrical about both the horizontal and vertical axes. The diagrams in the top row are in general bidirectional, while the bottom row has one bidirectional diagram corresponding to phase and space differences both equal to ½. It is of interest that for the most part cases where the phase and space separations are numerically equal correspond to unidirectional diagrams. However, these diagrams are only moderately sharp and thus far such arrays have not been used extensively in practice.

Referring again to the diagrams in the top row corresponding to 16 antennas all driven in phase, we note that directivity becomes progressively sharper as the spacing is increased until in the vicinity of $15/16\lambda$ appendages develop which soon surpass in magnitude the desired lobes. This effect is present in the commercial array, and limits, as we shall later see, the gain that may be derived from a given number of elements. The diagrams shown in Fig. 4 for 16 antennas are typical of others where the number of antennas in linear array is fairly large.

## The Linear Array and Reflector

One type of array now in commercial use consists of two parallel linear arrays of equiphased elements where the two parallel arrays are spaced ¼ wave-length and differ in relative phase by ¼ period. It is convenient to regard such a device either as two independent linear arrays, each having a directional characteristic as shown in the top row of Fig. 4, or as an array of couplets, each couplet of which has by itself a heart-shaped characteristic. Both antennas of the couplet may be independently driven at their prescribed phase separation of ¼ period, or one may derive its power from that radiated by the other, in which case the proper phase relation is automatically approximated [4] and the same practical result is obtained. In the latter case one is

[4] The problem of the reflecting antenna has been considered by Wilmotte and McPetrie, *Jour. I. E. E.*, **66**, 949, Englund and Crawford, *Proc. I. R. E.*, **17**, 1277; August, 1928, and Palmer and Honeyball, *Jour. I. E. E.*, **67**, 1045. Their conclusions indicate that the optimum separation between a single antenna and its reflector to give maximum forward radiation is roughly $\lambda/3$. However, it appears that when several antennas and reflectors are involved a separation more nearly $\lambda/4$ is optimum.

frequently known as the driven antenna and the other the reflector. This viewpoint is perhaps only a convenience and may not be altogether correct. An array of the above type transmits and receives best in a direction at right angles to its principal dimension. This type is, therefore, frequently known as a broadside array.

## Directive Diagrams from Arrays and Reflectors

In Fig. 5 is plotted a series of diagrams in a bisecting plane normal to the axis of each antenna of the array for different broadside arrangements such as are used commercially. They are systematically arranged horizontally in the order of the number of couplets in the array, and vertically with the increased spacing between adjacent couplets.

Several different forms of such directive diagrams are possible, which may be plotted in either polar or rectangular coordinates. In one form all diagrams are roughly of constant area and relative gains from various antenna systems are expressed in terms of the principal radius vector. In the second form the length of the principal radius vector remains constant and the relative gain is roughly inversely proportional to the area of the diagram. The second of these forms has been adopted in this paper largely because of the relative simplicity of the equation of the diagram and the facility with which properties of antennas may be determined.

In the lower left-hand corner of Fig. 5 will be found a plan showing the arrangement of the elements relative to the important direction of transmission. At its right is the general equation of these diagrams. This formula is also given as equation (14) of the appendix where the analytical theory of arrays is developed. Below each diagram is the ratio of the area of the circumscribed unit circle to the area of the horizontal diagram. Here also will be found the ratio of the area of the subordinate loops to the area of the main loop. The total area may be measured approximately with a planimeter or calculated more accurately by equation (32) in the mathematical appendix. In making up Fig. 5 each diagram was accurately plotted on standard polar coordinate paper from perhaps a hundred calculated points. This was then reduced photographically and the several diagrams were assembled.[5]

Inspection of the diagrams shows that increasing the number of couplets increases in all cases the sharpness of the main loop and hence the gain of the array. However, increasing the separation be-

[5] The diagrams used in this paper were calculated by a group of the Department of Development and Research of the American Telephone and Telegraph Company, under the direction of Miss E. M. Baldwin. Most of the material was checked by Mrs. Isabel Bemis, who assembled it in its present form and prepared the attached bibliography.

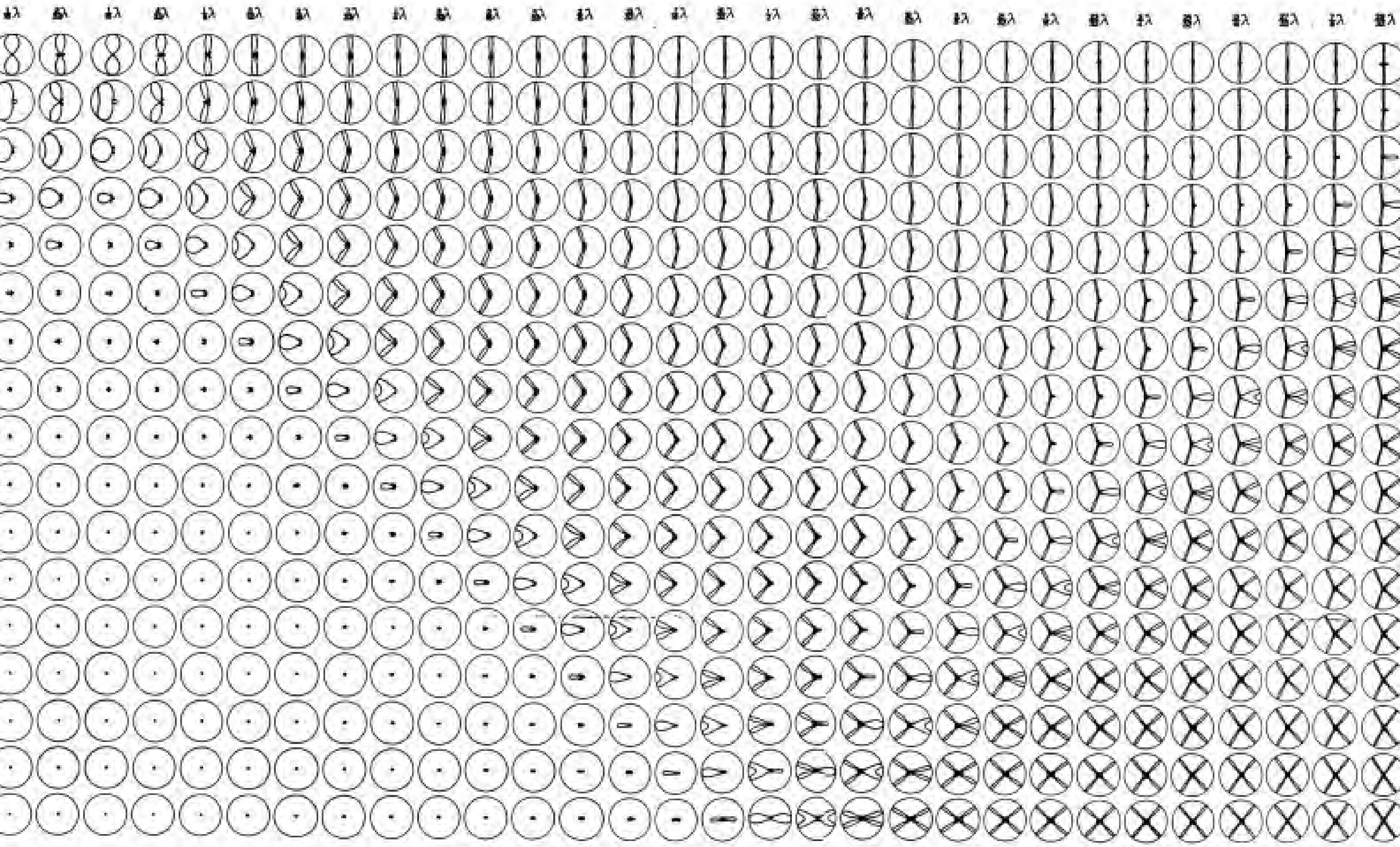


Fig. 6—Directive amplitude diagrams for an array of sixteen antennas. Separation in wave-lengths (λ) along the top. Phase difference in periods (T) at the left.

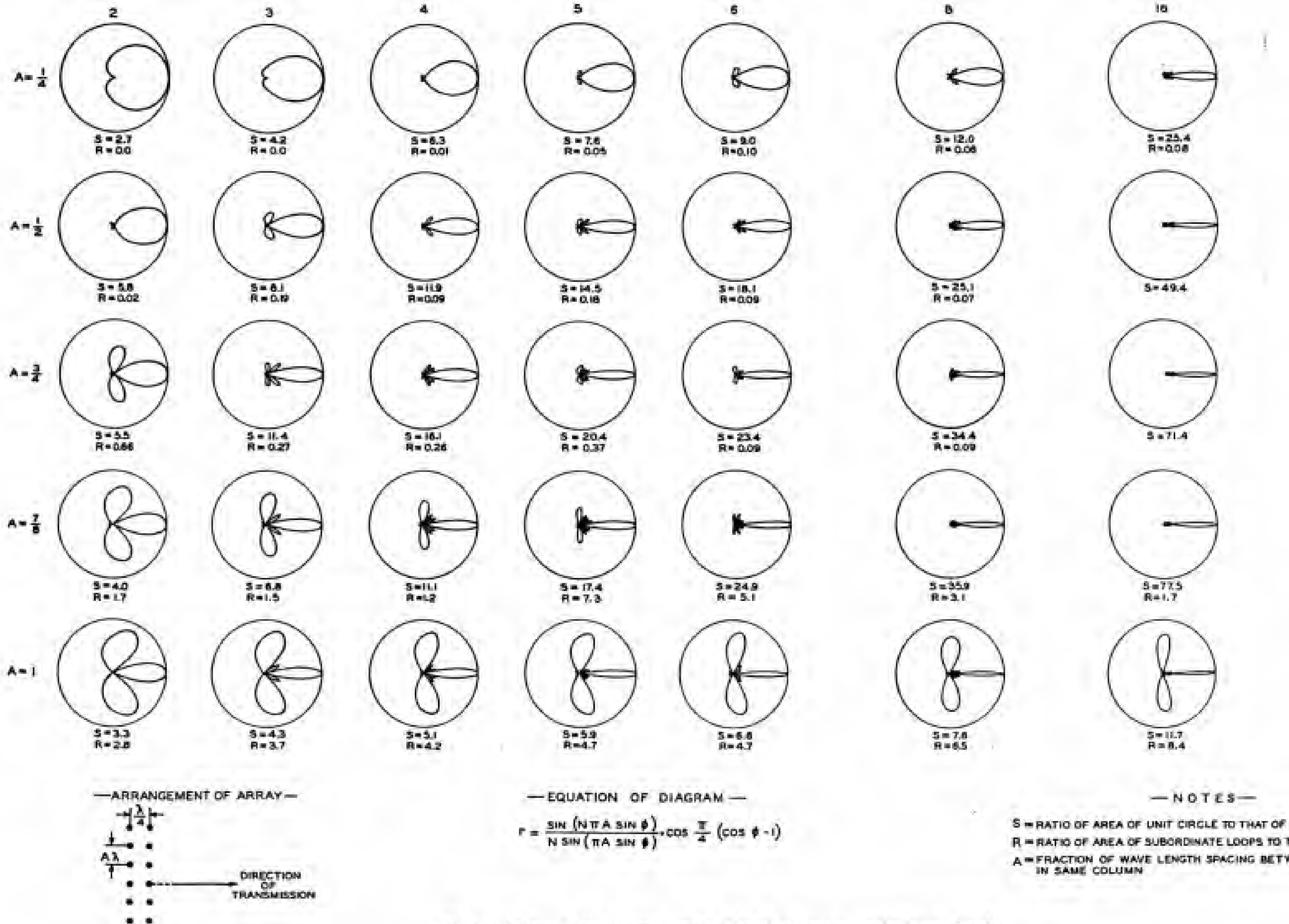



Fig. 5—Horizontal plane diagrams—number of couplets versus separation in wave-lengths.

tween couplets increases the gain only up to a certain point, after which the formation of parasitic lobes decreases the effectiveness of the array. The trend of these gains may be illustrated more effectively in graphical form.

In Fig. 6 calculated gain ratio is plotted against number of couplets giving one graph for each separation considered. These ratios are not based on the data given in Fig. 5, but were obtained from the integration of the equation of the directional diagram over an arbitrary sphere by use of equation (27) below. It may be noted that for many conditions the difference between these methods of calculating gain is only moderate. These power ratios are for the most part linear,

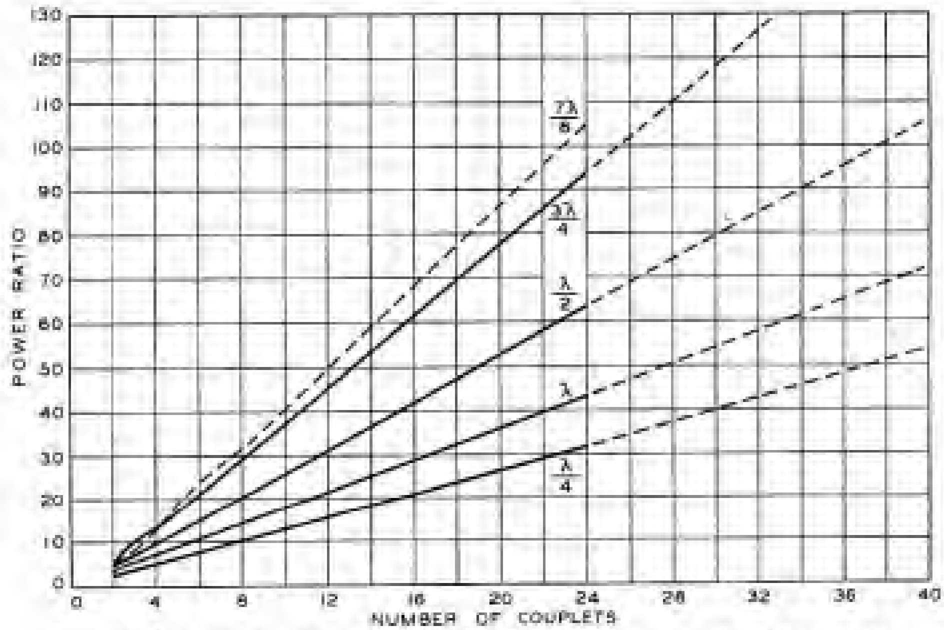



Fig. 6—Antenna arrays. Calculated power ratios vs. number of couplets.

indicating that such gains are proportional to the length of the array. This is in keeping with the view that a receiving antenna can intercept wave power more or less in proportion to its dimensions. It is also interesting to note that the slope of the curve of $\lambda/2$ is approximately twice that for $\lambda/4$, so that 16 couplets spaced $\frac{1}{4}$ wave-length give approximately the same gain as eight couplets spaced $\frac{1}{2}$ wave-length. This again shows that the length of the array is the most important criterion in determining its gain. In Fig. 7 the same data have been plotted in decibels.

In Fig. 8 gains expressed in decibels are plotted against the separation between elements. This shows more definitely the trend of the

antenna gain to a maximum, after which spurious lobes become of importance. Fig. 8 suggests that the spacing, giving optimum gain, would be the desideratum in antenna design. However, this is not

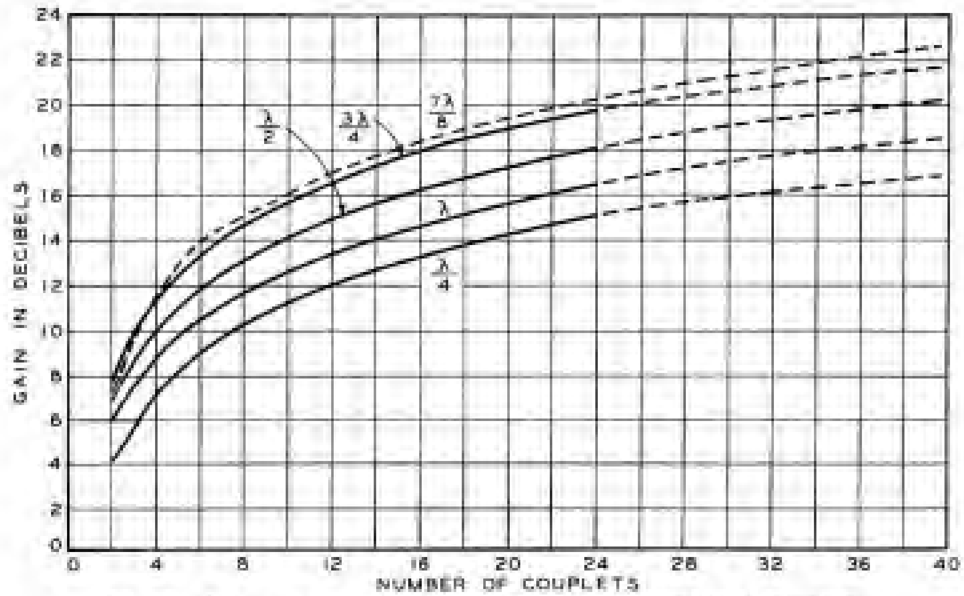



Fig. 7—Antenna arrays. Calculated gains vs. number of couplets.

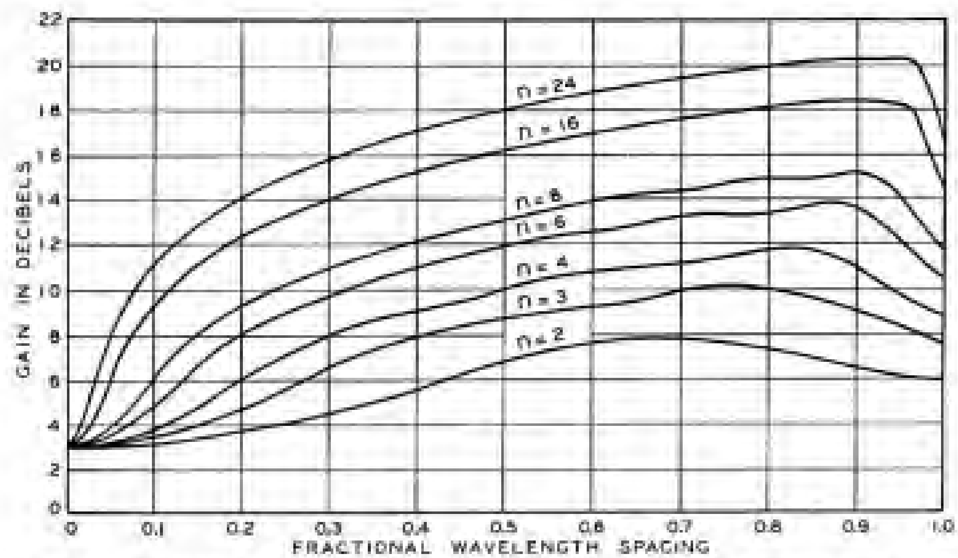



Fig. 8—Antenna arrays. Calculated gains vs. lateral spacing between couplets.

necessarily the case, as we shall presently see. It has already been pointed out that the over-all length of array, rather than the spacing or the number of conductors per unit length, constitutes the most

important factor in determining the gain. Furthermore, minimum area diagrams are frequently attended by fairly large spurious lobes which are undesirable particularly on receiving antennas. Also the

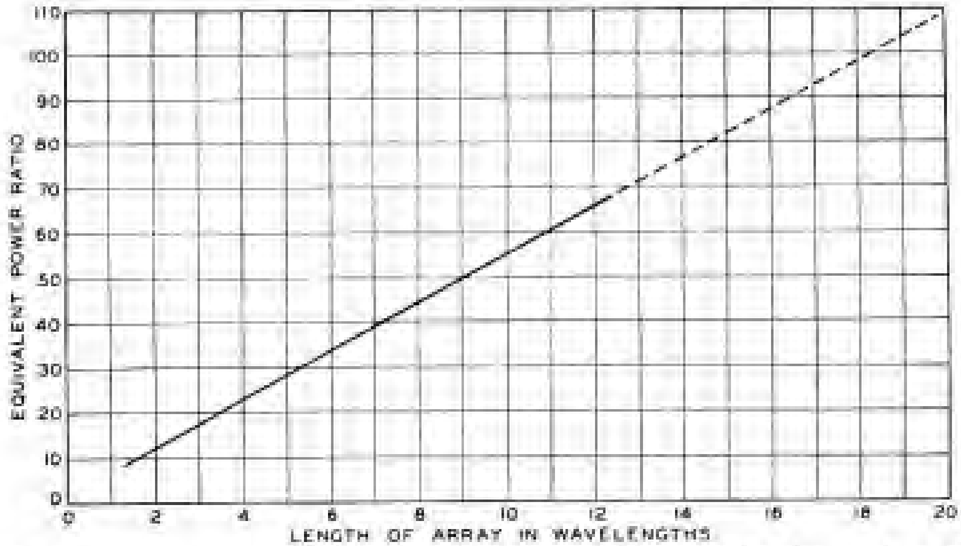



Fig. 9—Approximate gains to be expected from arrays of couplets for spacings of approximately λ/4 and λ/2.

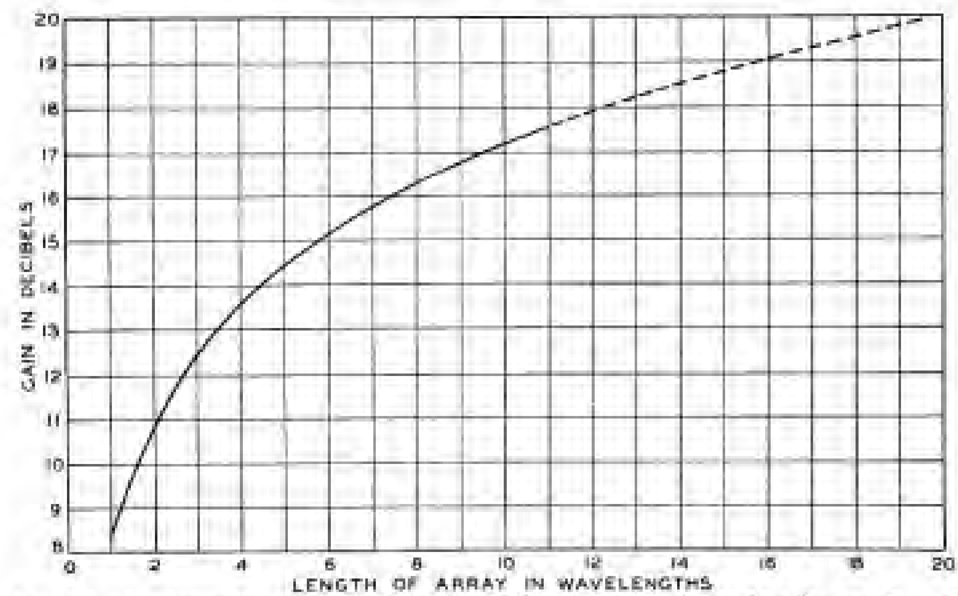



Fig. 10—Approximate gains to be expected from arrays of couplets for spacings of approximately λ/4 and λ/2.

cost of an antenna system of a given height is more or less proportional to its length, and in many cases is not materially affected by the number of conductors present. These considerations, together with the fact

that proper phases may often be most readily accomplished with intervals of either ¼ wave-length or ½ wave-length, have led to a rather general adoption of these closer spacings.

In Fig. 9, approximate gain ratios from arrays of various lengths have been plotted. These are most applicable for separations in the vicinity of ¼ and ½ wave-length. Fig. 10 shows the same data plotted in decibels. Within these limits, it appears that the gain ratio may be expressed by the simple formula $G = KL$, where $L$ is the array length in wave-lengths and $K$ is approximately 5.6. The result expressed in decibels is $G' = 10 \log_{10}(KL)$.

## Measured Antenna Gains

The degree to which the gains calculated above are approximated in practice is indicated by the data given in the diagrams of Figs. 11 and 12 and in Table I.

TABLE I

| Array Designation | Nominal Operating Frequency Megacycles | Number Couplets | Spacing | Measured Gain Over Similar Single Element db | Calculated Gain db | Difference db |
|---|---|---|---|---|---|---|
| 1-A..... | 18 | 24 | $\lambda/4$ | 15.3 | 15.0 | + 0.3 |
| 2-A..... | 18 | 24 | $\lambda/4$ | 15.2 | 15.0 | + 0.2 |
| 3-A..... | 18 | 24 | $\lambda/4$ | 15.0 | 15.0 | 0.0 |
| 1-B..... | 12 | 24 | $\lambda/4$ | 15.6 | 15.0 | + 0.6 |
| 2-B..... | 12 | 24 | $\lambda/4$ | 14.5 | 15.0 | − 0.5 |
| 3-B..... | 15 | 24 | $\lambda/4$ | 13.6 | 15.0 | − 1.4 |
| 4-B..... | 15 | 24 | $\lambda/4$ | 16.6 | 15.0 | + 1.6 |
| 2-C..... | 10 | 24 | $\lambda/4$ | 16.3 | 15.0 | + 1.3 |
| 3-C..... | 10 | 24 | $\lambda/4$ | 15.5 | 15.0 | + 0.5 |
| 1-C..... | 9 | 18 | $\lambda/4$ | 13.6 | 13.8 | − 0.2 |
| D *.... | 14 | 9 | $\lambda/2$ | 13.0 | 13.7 | − 0.7 |

* This antenna actually consisted of two arrays of four couplets each spaced laterally by one wave-length. The resultant diagram of such an array is for all practical purposes the same as that produced by a continuous array of nine couplets.

Fig. 11 shows a calculated diagram corresponding to certain receiving arrays used in the transatlantic telephone service between America and England. Several points are plotted on this diagram which correspond to the relative strengths of signals received at various angles. These points were obtained by observing the relative received signal voltage, measured on a standard field-strength measuring set connected to the array as an electric oscillator of constant amplitude was carried around the array at a distance of perhaps 20 wave-lengths. The plotted data correspond to the case where the

reflector was "floating." Although this arrangement most nearly corresponds to the conditions assumed in the calculated curve, it is not necessarily the most desirable adjustment to minimize noise arriving from the rear. This diagram corresponds to the antennas designated as 1-A, 2-A, and 3-A in Table I. These antennas consist effectively of 24 vertical couplets spaced horizontally at intervals of ¼ wave-length.

In this table are given further data on the strength of signals received on arrays, as compared with those received simultaneously on a single element of similar structure and height above earth. The different antennas represented involve varying conditions of wave-

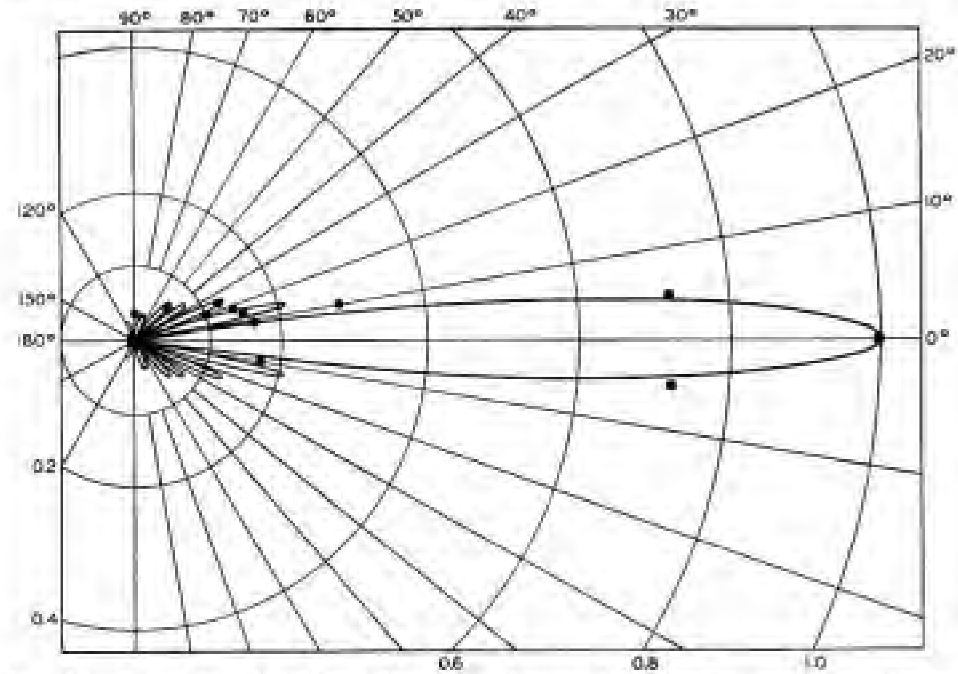



Fig. 11—Calculated directional diagram. Twenty-four couplets spaced one-fourth wave-length. Circles indicate experimental points.

length, height above earth, adjacent terrain, and types of support. These details are not believed to be of sufficient importance for discussion here. Two different array lengths are represented. The relative gains were substantially the same when observed on a local source of waves and when the signal came from a distant station. The last array represented in Table I was one used for transmitting. To effect the test, equal power was transmitted alternately from the array and from a single element while comparative measurements of electric field strength were made at a distance of approximately 3500 miles. The datum given is the mean of perhaps 100 observations extending over a total of eight hours on three different days. Two errors are

involved in the data of Table I. One is due to the doubtful magnitude of a correction necessary to account for the various heights at which the arrays were located above the earth and the second is the error of measurement of gain as compared with the reference antenna. These errors are approximately equal and together amount to ± 1 db.

In order to test further the agreement between measured gains and those calculated from the simple assumptions above, a receiving array was assembled step by step and corresponding measurements made. Certain precautions, such as to maintain impedance matches at points of coupling, were observed. The resulting data were plotted as points in Fig. 12. A smooth curve represents the corresponding calculated

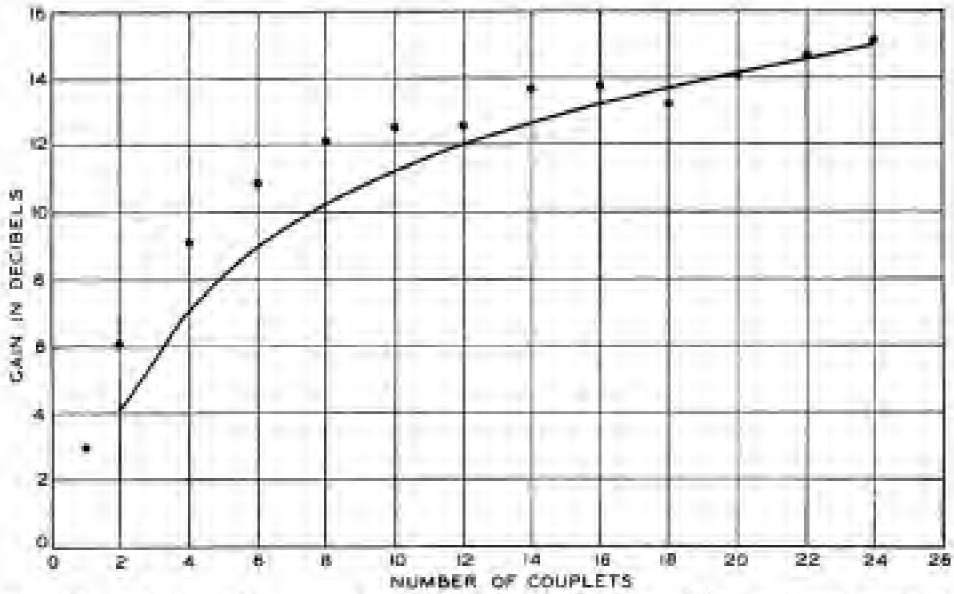



Fig. 12—Relation of measured to calculated gain of receiving antenna array at 14,350 kc.

data. It will be observed that the measured values are consistently higher than those calculated at the lower end of the curve, and in this region the agreement can hardly be regarded as satisfactory. However, limited time prevented a thorough study of the errors of measurement. Consequently these limited data may not be regarded as any adequate test of the theory.

## Combinations of Arrays

It may be shown that two or more similar directive systems may be combined to give a total directive effect, represented by the product of the individual effect, multiplied by the group effect. This principle is partially covered by equation (35) of the mathematical appendix.

Two cases are of special interest. First, it is sometimes desirable to divide an array into two or more bays, in order to make room for a supporting structure. This, of course, gives rise to a definite discontinuity in the over-all array.

Fig. 13 shows a series of diagrams resulting from a typical case of two such arrays, each having a length of 2½ wave-lengths but separated variously from 0 to 2 wave-lengths in steps as noted. These diagrams, of course, do not take into consideration the reaction resulting from proximity to an antenna mast, located in such an opening. The most important result is to emphasize the spurious lobes, as the spacing between arrays is increased.

A second effect of grouping which is of considerable interest is that of varying the direction of transmission by altering the respective phases between two or more arrays or between sections of the same array. In Fig. 14 a series of diagrams is shown for a typical case of two 3½ wave-length arrays, spaced one wave-length. All elements in the same array are driven in phase, but the two arrays differ in phase by various amounts, as noted. It will be observed that the possible rotational effect is very limited. The general equation for this diagram is given by formula (36) of the mathematical appendix.

This effect was investigated further by assuming a continuous array 7½ wave-lengths long, made up of 16 couplets spaced at intervals of ½ wave-length. The results are depicted in Fig. 15. The top row assumes that the array is divided into two sections of eight couplets each. This gives similar but not exactly the same results as those of Fig. 14. The array, however, might have been divided into other sections for purposes of phasing. The various possible combinations are tabulated below:

| Number of Sections | Number of Couplets per Section |
|---|---|
| 2 | 8 |
| 4 | 4 |
| 8 | 2 |
| 16 | 1 |

Diagrams in rows two, three, and four show that, as the array continues to be divided into smaller sections, the direction of transmission is capable of greater variation without sensible loss of sharpness. If the array be divided into two sections this range is limited to perhaps 3 deg. as in the case depicted in Fig. 14. Although this is very moderate, it is extremely useful in correcting for any errors in the orientation of the supporting structure or possibly correcting for deviation of the projected radiation caused by peculiarities of the adjacent terrain.

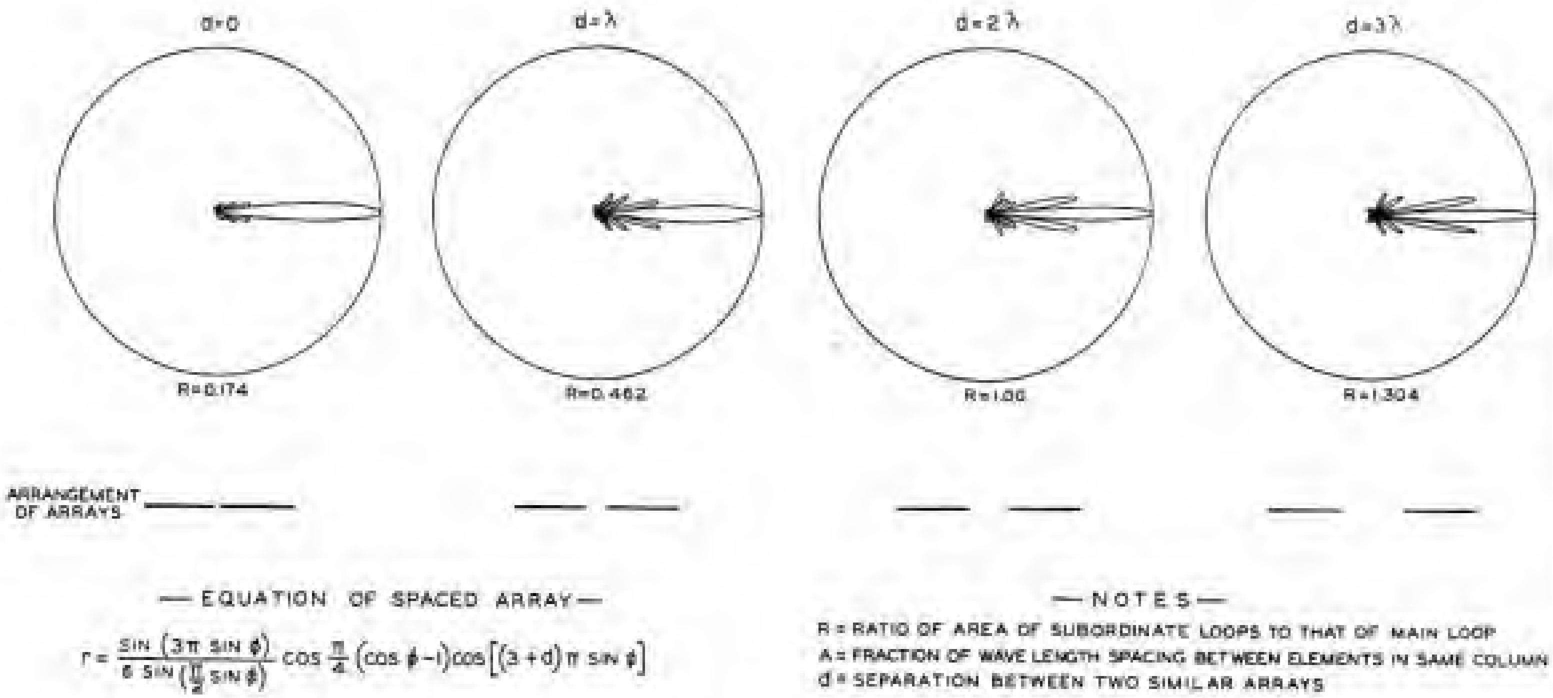



Fig. 13—Directional diagrams in horizontal plane of two arrays spaced laterally as noted. Case $N = 6$ and $A = \frac{1}{2}$.

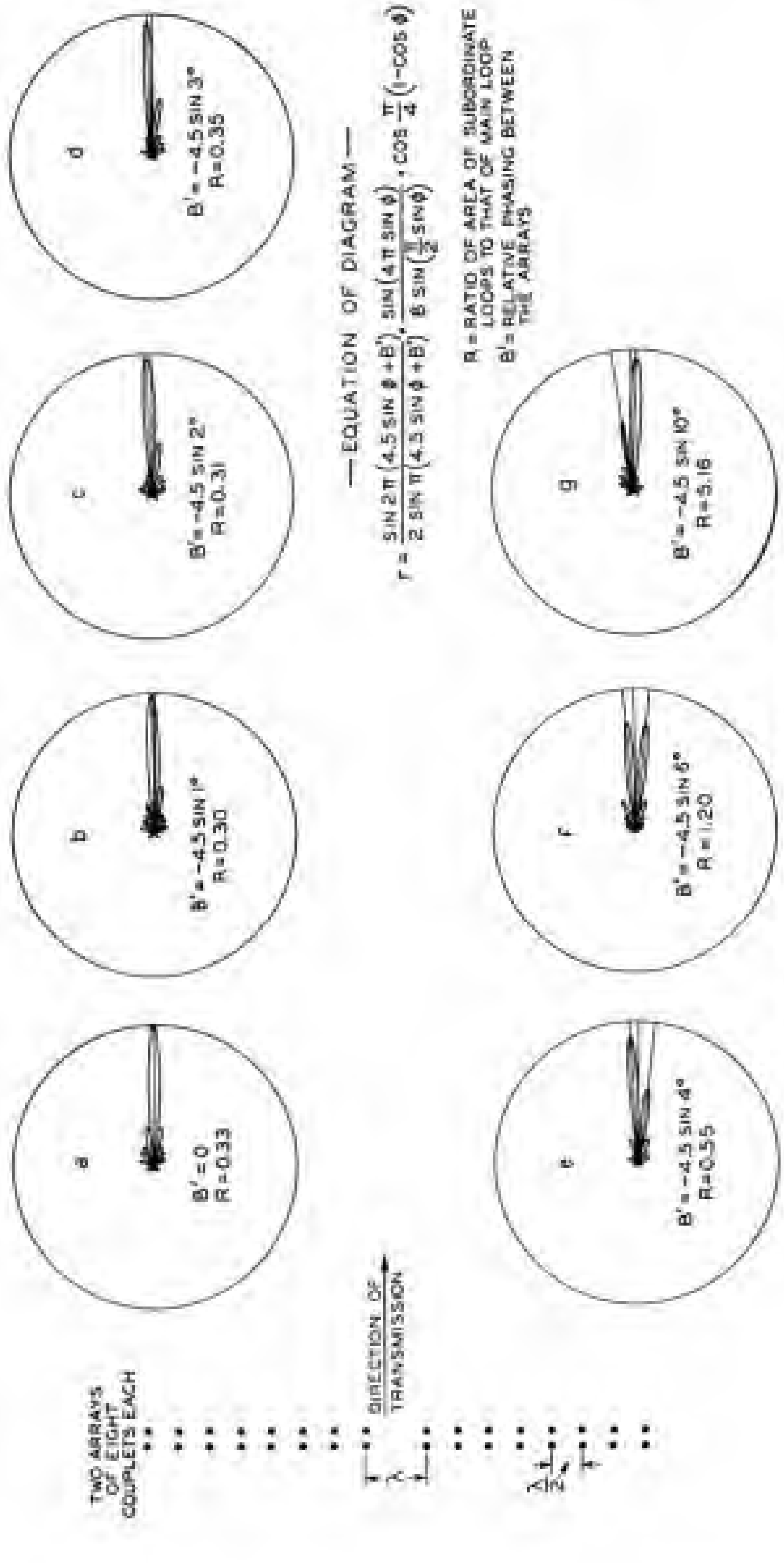



Fig. 14—Effect of phasing between two arrays.

If the array is divided into four sections the rotation may extend over a range of perhaps 9 deg., while for eight sections it may be 15 deg. The final case of 16 sections of one couplet each permits of considerable flexibility such as would be useful in operating with several distant stations in the same general direction. It should be pointed out, however, that the problem of making 16 phase adjustments each time a station wishes to change its direction of transmission is of considerable magnitude. For the particular case illustrated above it appears that the maximum rotation of the projected radiation is more or less proportional to the number of sections into which the array is divided. It may readily be seen from the two top rows of diagrams in Fig. 15 that continued addition of phasing amounts effectively to negative rotation. This may also be seen from an analysis of the equation of the diagram.

## Fields of Linear Arrays

The successful use of an array of couplets to give unidirectivity suggests that the use of more than two parallel linear arrays might further be employed to advantage.[6] Obviously many such combinations are possible, but one of some interest has been investigated below. As a concrete example of this variation of gain with arrangement of arrays, a series of diagrams for 36 elements has been plotted in Fig. 16. The condition of spacing and phase intervals between columns of each of $\frac{1}{4}\lambda$ has been chosen. The horizontal characteristic is given for separations between rows of both $\frac{1}{2}$ and $\frac{1}{4}$ wavelength. The vertical characteristic common to these two separations is also shown. The equation of the diagram is given in formula (17) of the mathematical appendix below.

It will be observed from Fig. 16 that the horizontal directivity is for the most part only moderate, but approaches a maximum for the condition where a long broadside array prevails, whereas the vertical directivity is increased by increasing the number of columns in the field. A substantial loop will be found near the rear of diagrams corresponding to an odd number of columns. It is of further interest that, as far as horizontal directivity alone is concerned, the optimum may be derived either from a single array of 36 elements or from 18 couplets. Considerations of both minimum interference and total gain, however, make the latter preferable. These conclusions may also be reached by more direct analysis.[7]

[6] U. S. Patent 1,643,323, John Stone Stone, September 27, 1927.

[7] Wilmotte, "General considerations of the directivity of beam systems," *Jour. I. E. E.*, **66**, 955.

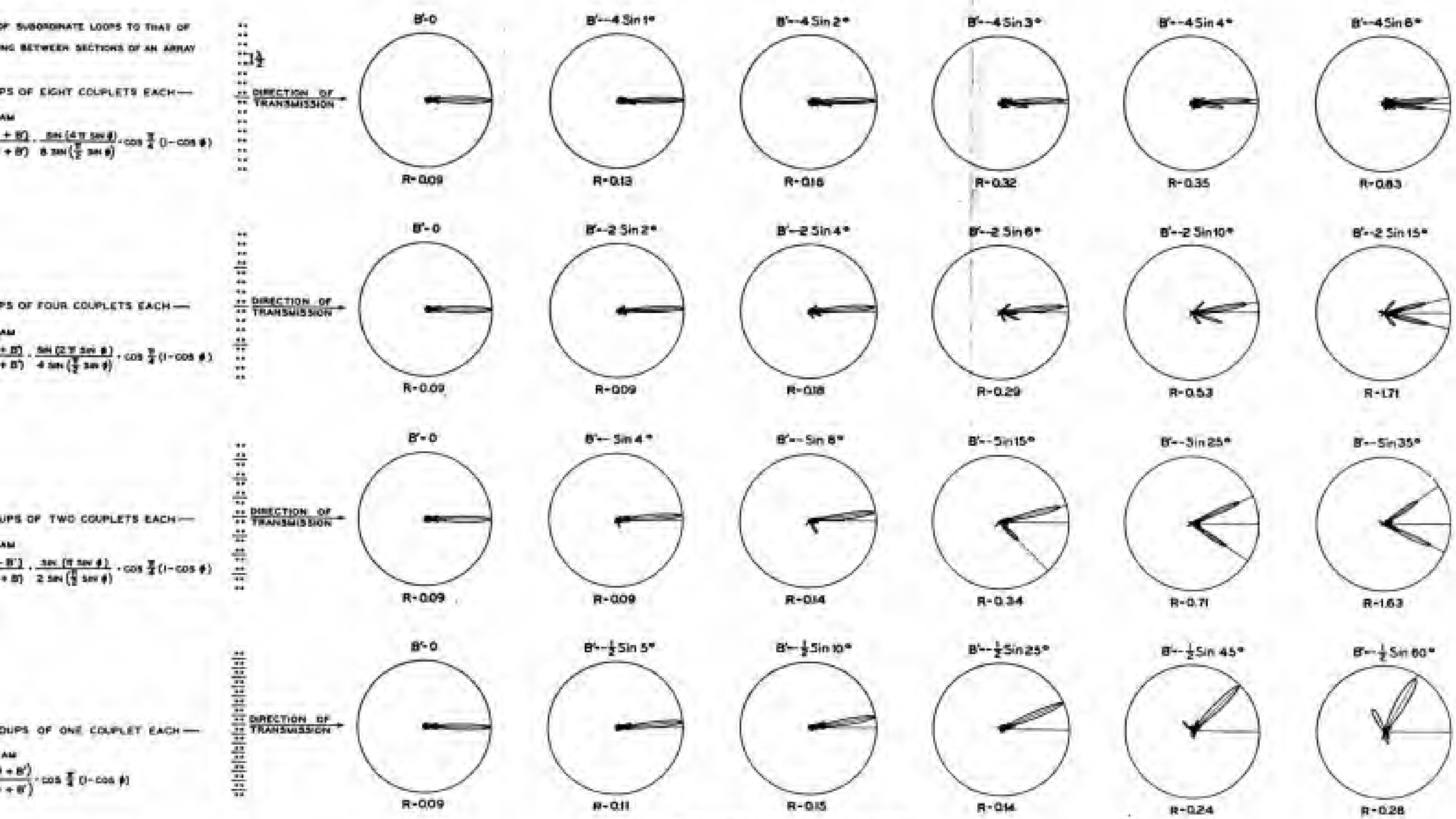



Fig. 15—Effect of phasing between sections of an array.

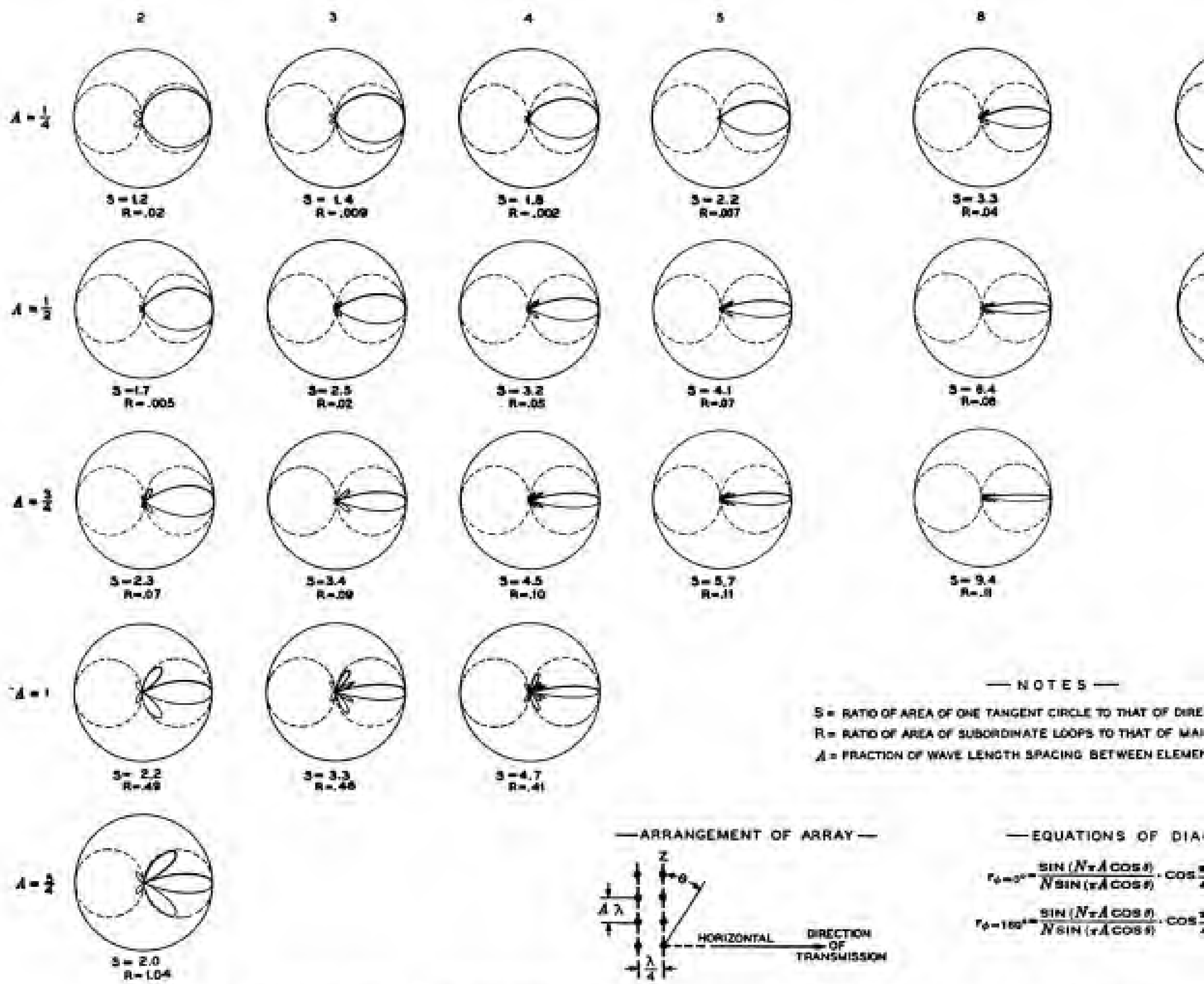



Fig. 17—Vertical plane diagrams due to couplets of coaxial antennas—number of couplets versus separation in wave-lengths.

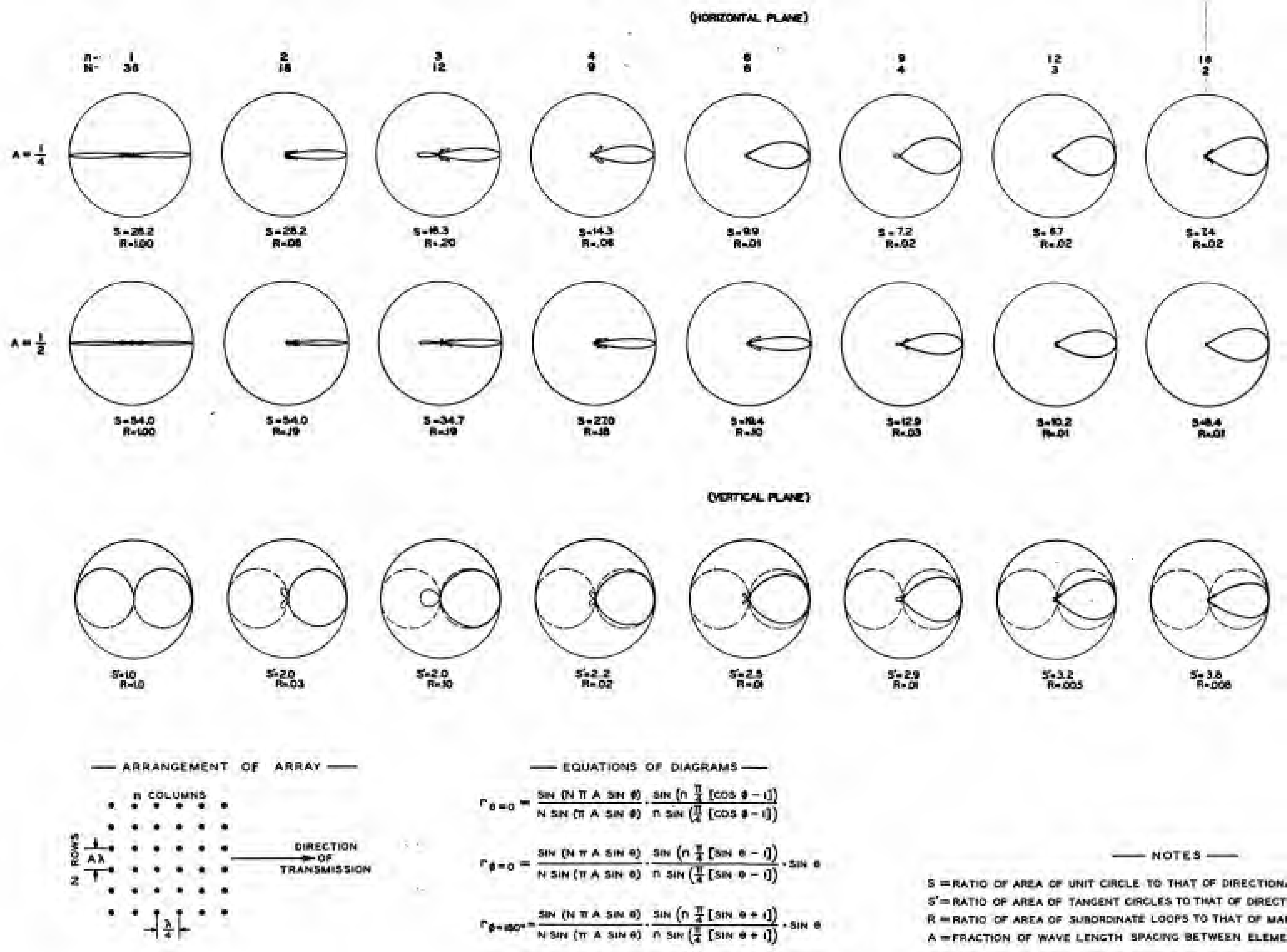



Fig. 16—Directional diagrams due to a field of thirty-six antennas.

## Stacked Antennas

Thus far the discussion has centered mainly around directivity produced by placing vertical antennas in horizontal array. Added gain may be had also by incorporating directivity in a vertical plane.[8] This is frequently accomplished by arranging individual antennas one above another with their axes collinear, and is sometimes known as stacking. The fundamental principles of analysis are the same as those already utilized. However, an approximate correction must be allowed to account for the fact that the radiation from a linear oscillator increases from zero along the axis to a maximum in a plane perpendicular to the axis. The directional characteristic in planes passed through and parallel to such a radiator is approximated by two tangent circles.

Fig. 17 shows a series of directional diagrams indicating the results of stacking unidirectional couplets. The diagrams shown refer to the plane passed through the axes of the two linear oscillators comprising the couplet. On each diagram is a unit circle corresponding to a single point source. Inscribed are the two tangent circles, representing the vertical directional characteristic of a single linear source. Inside one of the tangent circles is the final directional diagram of the stacked array. The ratio of the area of the tangent circles to that of the characteristic diagram is given under each figure. This may be regarded as a rough measure of the relative gain. These diagrams are arranged horizontally in order of increasing number of couplets and vertically in order of separation. It frequently happens in practice that each radiator is approximately ½ wave-length long so it is convenient to utilize a vertical spacing interval also of ½ wave-length. Consequently the second row of diagrams is probably of greatest practical interest. In calculating these diagrams earth effects have been ignored.

In Figs. 18 and 19, the gain in decibels to be expected from stacking couplets has been plotted against number of couplets and fractional wave-length spacing. These values, like those for Figs. 7 and 8 above, were calculated by integrating the equation of diagram over a sphere of arbitrary radius. This was accomplished by use of equation (30) below. On account of the limited data at hand, Figs. 18 and 19 should be regarded only as a convenient method of illustrating the trend of the variables. These indicate that somewhat lower corresponding improvements result from stacking than from increasing the length of an array.

[8] U. S. Patent 1,683,739, John Stone Stone, September 11, 1928.

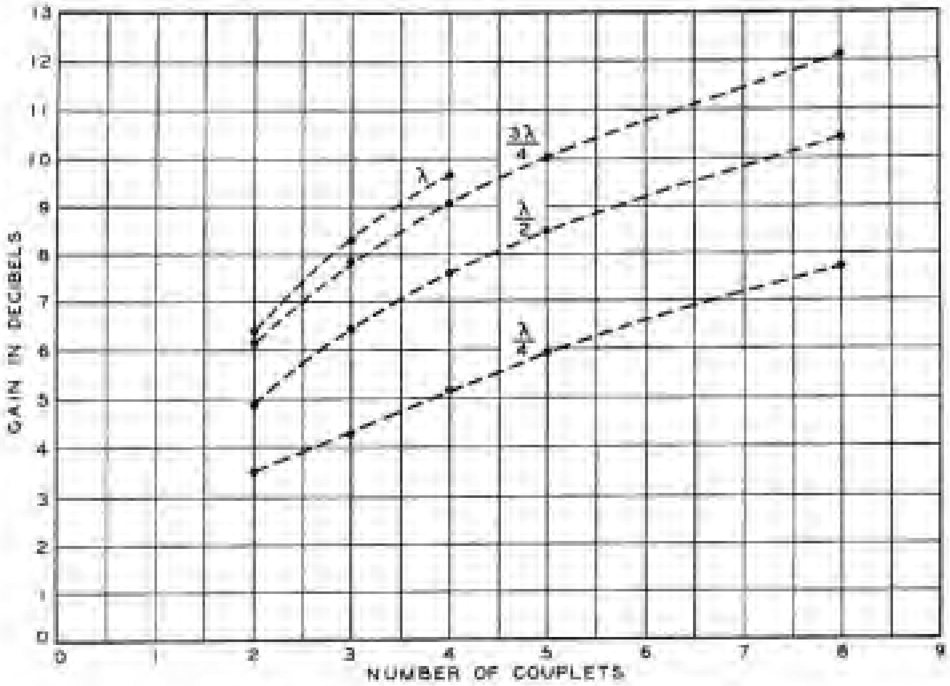



Fig. 18—Calculated gains from stacked antennas.

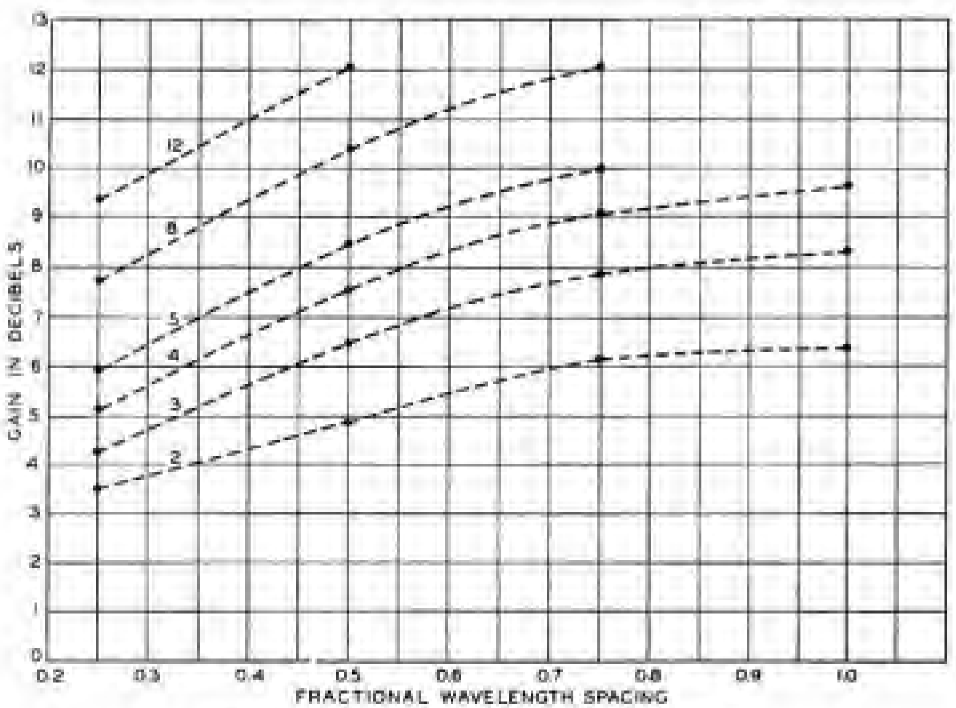



Fig. 19—Calculated gains from stacked antennas.

## Arrays Incorporating Both Horizontal and Vertical Directivity

The gains of arrays combining both horizontal and vertical directivity may not be simply calculated by adding the gains (expressed in decibels) corresponding to elements arranged respectively along the two principal coordinate axes. However, they may be calculated except for earth effects by means of equation (26) below. Some calculations of this kind have been made and the data are tabulated below. They assume a total of 36 couplets which are arranged variously as noted. In the first case all 36 couplets are arranged as a simple horizontal array. The second case assumes that they are arranged in a

TABLE II

| Number of Couplets Along Horizontal Axis | Number of Couplets Along Vertical Axis | Gain over Single Half-Wave Element Decibels |
|---|---|---|
| *N* | *N* | *G* |
| 36 | 1 | 19.7 |
| 18 | 2 | 19.0 |
| 12 | 3 | 18.9 |
| 9 | 4 | 18.8 |
| 6 | 6 | 18.7 |
| 4 | 9 | 18.6 |
| 1 | 36 | 17.5 |

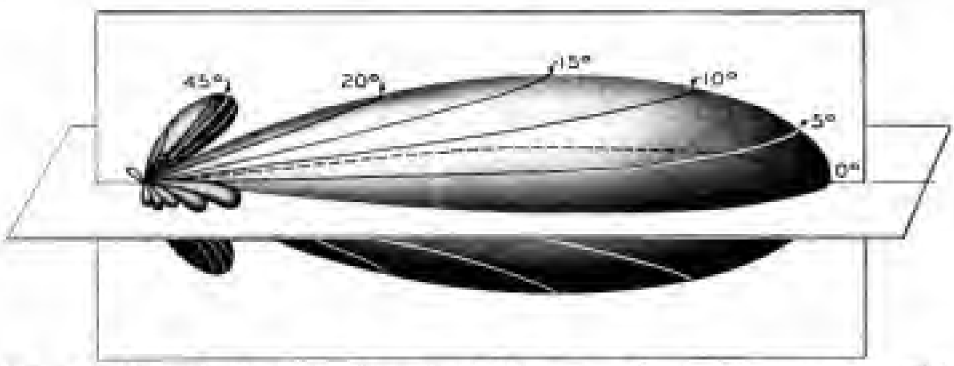



Fig. 20—Approximate three-dimensional diagram. Linear antenna array with reflector. Aperture two wave-lengths by eight wave-lengths.

broadside rectangle two elements high and 18 elements wide. This combination may be regarded as two arrays of 18 couplets arranged one above the other. The third case similarly assumes three arrays of 12 couplets each. A separation between couplets of ½ wave-length has been assumed throughout. The most economical arrangement of such an array depends not only on the relative costs of real estate and towers but also on feed-line losses and effects due to the proximity

of the earth. The latter have specifically been omitted in this discussion.

Fig. 20 shows roughly the calculated directional characteristics of a typical stacked array incorporating both horizontal and vertical directivity. The planes passed through the diagram serve only as convenient references to assist in visualizing the horizontal and vertical diagrams. Earth effects of course, have been ignored.

## Appendix

A general case of linear arrays which includes those used extensively in short-wave radio work, consists of a number of sources equispaced and equiphased along each of the three principal coordinate axes such that the space between sources is made up of rectangular parallelopipeds with the individual sources located at each corner. This may be regarded as $\boldsymbol{N}$ parallel planes each made up of $N$ parallel columns where each column is made up of $n$ individual radiating elements. The arrangement is made more evident by Fig. 21. The

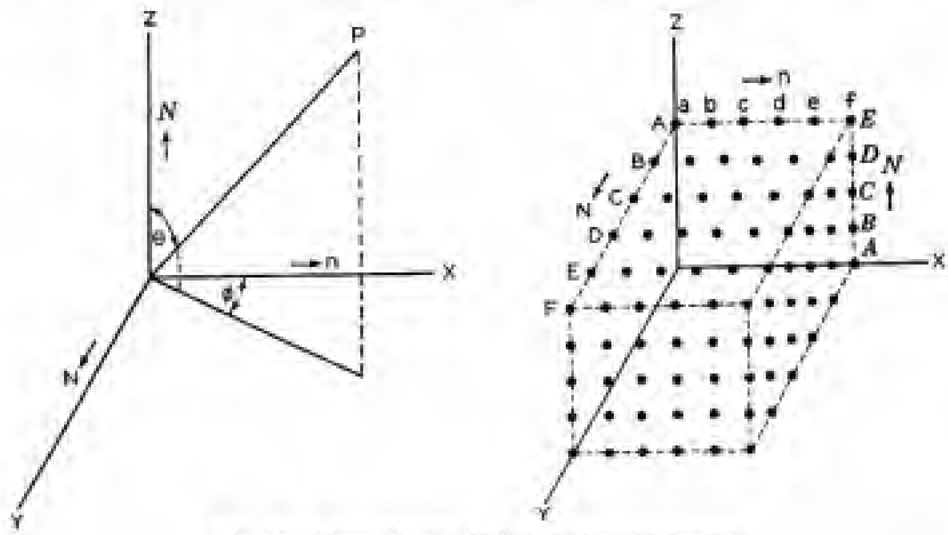



Fig. 21—General case of linear antenna arrays.

usual conventions for representing three-dimensional space have been adopted. We may designate the spacing between elements along the $x$, $y$, and $z$ axes, respectively, by $a\lambda$, $A\lambda$, and $\boldsymbol{A}\lambda$ and their corresponding phase displacements between adjacent elements along the three principal axes by $bT$, $BT$ and $\boldsymbol{B}T$.

The distance from any point in space to a particular radiator is

$$R_{nN\boldsymbol{N}} \doteq R - (\boldsymbol{N} - 1)\boldsymbol{A}\lambda \cos \theta - (N - 1)A\lambda \cos \phi \sin \theta - (n - 1)a\lambda \sin \theta \sin \phi. \tag{1}$$

Similarly the time phase of any particular element relative to the origin is

$$\delta_{nN\boldsymbol{N}} = [(n-1)b + (N-1)B + (\boldsymbol{N}-1)\boldsymbol{B}]T. \tag{2}$$

The instantaneous value of the electric field at any remote point $P$ due to one of these sources is given by

$$E_{n'} = A \cos \frac{2\pi}{\lambda}(ct - R_{n'}) + \delta_{n'} = A \cos \psi_{n'}, \tag{3}$$

where $n' = nN$.

The resultant interfering effect at a point $P$ due to $n'$ such sources all of equal amplitude is given by

$$\begin{aligned} E^2 = n'E_0^2 + 2E_0^2[&\cos(\psi_1 - \psi_2) + \cos(\psi_1 - \psi_3) + \cos(\psi_1 - \psi_4) + \cdots \text{ etc.} \\ &+ \cos(\psi_2 - \psi_3) + \cos(\psi_2 - \psi_4) + \cos(\psi_2 - \psi_5) + \cdots \text{ etc.} \\ &+ \cos(\psi_3 - \psi_4) + \cos(\psi_3 - \psi_5) + \cdots \text{ etc.} \\ &+ \cos(\psi_{n'-1} - \psi_{n'})]. \end{aligned} \tag{4}$$

The summation above gives rise to three series as follows:

$$\begin{aligned} S_x = {} & (n-1)\cos 2\pi(a \sin\theta \cdot \sin\phi + b) \\ & + (n-2)\cos 2\cdot 2\pi(a \sin\theta \cdot \sin\phi + b) \\ & + (n-3)\cos 3\cdot 2\pi(a \sin\theta \cdot \sin\phi + b) + \cdots \\ & + \cos(n-1)\cdot 2\pi(a \sin\theta \cdot \sin\phi + b), \end{aligned} \tag{5}$$

$$\begin{aligned} S_y = {} & (N-1)\cos 2\pi(A \sin\theta \cdot \cos\phi + B) \\ & + (N-2)\cos 2\cdot 2\pi(A \sin\theta \cdot \cos\phi + B) \\ & + (N-3)\cos 3\cdot 2\pi(A \sin\theta \cdot \cos\phi + B) + \cdots \\ & + \cos(N-1)\cdot 2\pi(A \sin\theta \cdot \cos\phi + B), \end{aligned} \tag{6}$$

$$\begin{aligned} S_z = {} & (\boldsymbol{N}-1)\cos 2\pi(\boldsymbol{A} \cos\theta + \boldsymbol{B}) \\ & + (\boldsymbol{N}-2)\cos 2\cdot 2\pi(\boldsymbol{A} \cos\theta + \boldsymbol{B}) \\ & + (\boldsymbol{N}-3)\cos 3\cdot 2\pi(\boldsymbol{A} \cos\theta + \boldsymbol{B}) + \cdots \\ & + \cos(\boldsymbol{N}-1)\cdot 2\pi(\boldsymbol{A} \cos\theta + \boldsymbol{B}), \end{aligned} \tag{7}$$

such that

$$E^2 = E_0^2(n + 2S_x)(N + 2S_y)(\boldsymbol{N} + 2S_z). \tag{8}$$

Each series is of the type

$$\begin{aligned} S = (n-1)\cos x + (n-2)\cos 2x \\ + (n-3)\cos 3x + \cdots + \cos(n-1)x' \end{aligned} \tag{9}$$

which is readily summed giving

$$n + 2S = \frac{(\cos nx - 1)}{(\cos x - 1)} = \frac{\sin^2 \frac{nx}{2}}{\sin^2 \frac{x}{2}}, \tag{10}$$

so

$$E = E_0 \frac{\sin n\pi(a \cos \phi \cdot \sin \theta + b)}{\sin \pi(a \cos \phi \cdot \sin \theta + b)} \cdot \frac{\sin N\pi(A \sin \phi \cdot \sin \theta + B)}{\sin \pi(A \sin \phi \cdot \sin \theta + B)} \cdot \frac{\sin N\pi(\boldsymbol{A} \cos \theta + \boldsymbol{B})}{\sin \pi(\boldsymbol{A} \cos \theta + \boldsymbol{B})} \cdot \tag{11}$$

Reducing to common voltage level and including a term $\sin \theta$ to cover the case of radiation from linear oscillators we have for the equation of the directional diagram

$$r = \frac{\sin n\pi(a \cos \phi \cdot \sin \theta + b)}{n \sin \pi(a \cos \phi \cdot \sin \theta + b)} \cdot \frac{\sin N\pi(A \sin \phi \cdot \sin \theta + B)}{N \sin \pi(A \sin \phi \cdot \sin \theta + B)} \cdot \frac{\sin \boldsymbol{N}\pi(\boldsymbol{A} \cos \theta + \boldsymbol{B})}{\boldsymbol{N} \sin \pi(\boldsymbol{A} \cos \theta + \boldsymbol{B})} \cdot \sin \theta. \tag{12}$$

It will be recognized that this equation is made up of four factors. The first three account for the effects of the disposition of elements along the $x$, $y$, and $z$ axes, respectively, while the fourth, of course, accounts for the direction of radiation from a linear oscillator. This is an equation giving magnitudes only. In plotting polar diagrams from this equation negative signs have no physical significance, and are plotted in a positive sense.

An examination of this equation shows that there are many possibilities which allow radiation in preferred directions, and at the same time limit it in others. Some of these are discussed below.

## Special Cases

If we assume $n = 2$, $a = \frac{1}{4}$, $b = -\frac{1}{4}$ and $B = \boldsymbol{B} = 0$

$$r = \frac{\sin (N\pi A \sin \phi \cdot \sin \theta)}{N \sin (\pi A \sin \phi \cdot \sin \theta)} \cdot \frac{\sin (\boldsymbol{N}\pi\boldsymbol{A} \cos \theta)}{\boldsymbol{N} \sin (\pi\boldsymbol{A} \cos \theta)} \cdot \cos \frac{\pi}{4} (\cos \phi \cdot \sin \theta - 1) \cdot \sin \theta. \tag{13}$$

This corresponds to the practical case of transmission along the $x$ axis from an antenna curtain and reflector made up of $N$ vertical columns of $\boldsymbol{N}$ elements each.

The equation for the diagram in the $(XY)$ plane may be had by placing $\theta = \pi/2$ giving

$$r = \frac{\sin (N\pi A \sin \phi)}{N \sin (\pi A \sin \phi)} \cos \frac{\pi}{4} (\cos \phi - 1), \tag{14}$$

which is the equation of the diagrams in Fig. 5 above. The corresponding equation for the principal vertical section may be had by placing $\phi = 0$ and $\phi = \pi$ giving

$$\left.\begin{aligned} r &= \frac{\sin (N\pi A \cos \theta)}{N \sin (\pi A \cos \theta)} \cos \frac{\pi}{4} (\sin \theta - 1) \sin \theta \\ \text{and} \qquad r &= \frac{\sin (N\pi A \cos \theta)}{N \sin (\pi A \cos \theta)} \cos \frac{\pi}{4} (\sin \theta + 1) \sin \theta \end{aligned}\right\}, \tag{15}$$

which is the equation for the diagrams of Fig. 17.

The diagram of a single linear array of point sources is specified by the first term of equation (12) where $\theta = \pi/2$ or

$$r = \frac{\sin n\pi(a \cos \phi + b)}{n \sin \pi(a \cos \phi + b)} \cdot \tag{16}$$

The diagrams of Figs. 3 and 4 above may be calculated from equation (16) by placing $n = 2$ and $n = 16$, respectively. This also agrees with Foster's equation (1), page 307.[2]

The diagram of a field of coplanar linear arrays such as depicted in Fig. 16 above follows from equation (12) by placing $N = 1$, $a = \frac{1}{4}$, $b = -\frac{1}{4}$ and $B = 0$.

If the diagram is to be restricted to the $(XY)$ plane, $\theta = \pi/2$ and

$$r = \frac{\sin (N\pi A \sin \phi)}{N \sin (\pi A \sin \phi)} \cdot \frac{\sin \left( n \frac{\pi}{4} (\cos \phi - 1) \right)}{n \sin \left( \frac{\pi}{4} (\cos \phi - 1) \right)} \cdot \tag{17}$$

## Calculated Gains from Arrays

The flow of power through each unit area due to an advancing electric wave is given by the Poynting vector as

$$s = \frac{c}{4\pi} E \times H, \tag{18}$$

where $E$ and $H$ are vectors representing respectively, the electric and magnetic components of the advancing wave.

[2] Loc. cit.

For free space $|E| = |H|$ so

$$s = \frac{c}{4\pi} E^2. \tag{19}$$

Now the total power radiated through a sphere enclosing an array of sources is

$$P_1 = \int s d\sigma = \frac{c}{4\pi} \int_0^{\pi} \int_0^{2\pi} E_1^2 \sin\theta d\phi d\theta. \tag{20}$$

A second system would give

$$P_2 = \frac{c}{4\pi} \int_0^{\pi} \int_0^{2\pi} E_2^2 \sin\theta d\phi d\theta. \tag{21}$$

The radiated powers of these two systems might be so adjusted at the source as to give equal fields at any point along a preferred direction. A ratio of these powers, therefore, would be a convenient measure of the relative directional properties of the two arrays. This "test ratio" may conveniently be set up in terms of the equations of the diagrams derived above. In which case

$$T = \frac{\int_0^{\pi} \int_0^{2\pi} r_1^2 \sin\theta d\phi d\theta}{\int_0^{\pi} \int_0^{2\pi} r_2^2 \sin\theta d\phi d\theta}. \tag{22}$$

If we assume all comparisons are to be made with respect to a single linear oscillator the denominator reduces to $8\pi/3$, so

$$T = \frac{3}{8\pi} \int_0^{\pi} \int_0^{2\pi} r_1^2 \sin\theta d\phi d\theta. \tag{23}$$

This ratio may conveniently be expressed in decibels. In which case $G = 10 \log_{10} 1/T$ is sometimes called the gain of an array.

If we are interested in the solid array shown in Fig. 21, where $n \cdot N \cdot \boldsymbol{N}$ linear oscillators, each having respective space and phase separations of $a\lambda$, $bT$; $A\lambda$, $BT$; and $\boldsymbol{A}\lambda$, $\boldsymbol{B}T$, are arranged progressively along the three principal coordinate axes, this becomes

$$\begin{aligned} T = \frac{3}{8\pi} \int_0^{\pi} \int_0^{2\pi} & \frac{\sin^2 [n\pi(a \cos\phi \sin\theta + b)]}{n^2 \sin^2 [\pi(a \cos\phi \sin\theta + b)]} \\ & \cdot \frac{\sin^2 [N\pi(A \sin\phi \sin\theta + B)]}{N^2 \sin^2 [\pi(A \sin\phi \sin\theta + B)]} \\ & \cdot \frac{\sin^2 [\boldsymbol{N}\pi(\boldsymbol{A} \cos\theta + \boldsymbol{B})]}{\boldsymbol{N}^2 \sin^2 [\pi(\boldsymbol{A} \cos\theta + \boldsymbol{B})]} \cdot \sin^3 \theta d\phi d\theta. \end{aligned} \tag{24}$$

This integration has been carried out by R. M. Foster who has very kindly placed the results at the writer's disposal. Only the final result is given herewith:

$$\begin{aligned}
T = \frac{1}{nN\boldsymbol{N}} &+ \frac{3}{n^2N\boldsymbol{N}}\sum_{k=1}^{n-1}(n-k)\cdot\cos(2\pi kb)\cdot Q(2\pi ka, 0) \\
&+ \frac{3}{nN^2\boldsymbol{N}}\sum_{K=1}^{N-1}(N-K)\cdot\cos(2\pi KB)\cdot Q(2\pi KA, 0) \\
&+ \frac{3}{nN\boldsymbol{N}^2}\sum_{\boldsymbol{K}=1}^{\boldsymbol{N}-1}(\boldsymbol{N}-\boldsymbol{K})\cdot\cos(2\pi\boldsymbol{KB})\cdot Q(0, 2\pi\boldsymbol{KA}) \\
&+ \frac{6}{n^2N^2\boldsymbol{N}}\sum_{k=1}^{n-1}\sum_{K=1}^{N-1}(n-k)(N-K)\cdot\cos(2\pi KB) \\
&\qquad\cdot\cos(2\pi kb)\cdot Q(2\pi\sqrt{k^2a^2+K^2A^2}, 0) \\
&+ \frac{6}{nN^2\boldsymbol{N}^2}\sum_{K=1}^{N-1}\sum_{\boldsymbol{K}=1}^{\boldsymbol{N}-1}(N-K)(\boldsymbol{N}-\boldsymbol{K})\cdot\cos(2\pi KB) \\
&\qquad\cdot\cos(2\pi\boldsymbol{KB})\cdot Q(2\pi KA, 2\pi\boldsymbol{KA}) \\
&+ \frac{6}{n^2N\boldsymbol{N}^2}\sum_{k=1}^{n-1}\sum_{\boldsymbol{K}=1}^{\boldsymbol{N}-1}(n-k)(\boldsymbol{N}-\boldsymbol{K})\cdot\cos(2\pi kb) \\
&\qquad\cdot\cos(2\pi\boldsymbol{KB})\cdot Q(2\pi ka, 2\pi\boldsymbol{KA}) \\
&+ \frac{12}{n^2N^2\boldsymbol{N}^2}\sum_{k=1}^{n-1}\sum_{K=1}^{N-1}\sum_{\boldsymbol{K}=1}^{\boldsymbol{N}-1}(n-k)(N-K)(\boldsymbol{N}-\boldsymbol{K}) \\
&\qquad\cdot\cos(2\pi kb)\cdot\cos(2\pi KB)\cdot\cos(2\pi\boldsymbol{KB}) \\
&\qquad\cdot Q(2\pi\sqrt{k^2a^2+K^2A^2}, 2\pi\boldsymbol{KA}). \qquad (25)
\end{aligned}$$

Where the function

$$\begin{aligned}
Q(x, y) = \frac{x^2}{(x^2+y^2)^{3/2}}\sin(\sqrt{x^2+y^2}) &+ \frac{x^2-2y^2}{(x^2+y^2)^2}\cos(\sqrt{x^2+y^2}) \\
&- \frac{x^2-2y^2}{(x^2+y^2)^{5/2}}\sin(\sqrt{x^2+y^2}). \qquad (25a)
\end{aligned}$$

In particular

$$Q(x, 0) = \frac{\sin x}{x} + \frac{\cos x}{x^2} - \frac{\sin x}{x^3} \qquad (25b)$$

and

$$Q(0, x) = -\frac{2\cos x}{x^2} + \frac{2\sin x}{x^3}, \qquad (25c)$$

## Special Cases

(1) If we assume $n = 2$, $a = \frac{1}{4}$, $b = -\frac{1}{4}$ and $B = \boldsymbol{B} = 0$, the test ratio is given by

$$
\begin{aligned}
T_1 = \frac{1}{2N\mathbf{N}} &+ \frac{3}{2N^2\mathbf{N}}\sum_{K=1}^{N-1}(N-K)\cdot Q(2\pi KA,\,0) \\
&+ \frac{3}{2N\mathbf{N}^2}\sum_{\mathbf{K}=1}^{\mathbf{N}-1}(\mathbf{N}-\mathbf{K})\cdot Q(0,\,2\pi\mathbf{KA}) \\
&+ \frac{3}{N^2\mathbf{N}^2}\sum_{K=1}^{N-1}\sum_{\mathbf{K}=1}^{\mathbf{N}-1}(N-K)(\mathbf{N}-\mathbf{K})\cdot Q(2\pi KA,\,2\pi\mathbf{KA}). \qquad (26)
\end{aligned}
$$

This, like equation (13), corresponds to the practical case of transmission from an antenna curtain and reflector each made up of $N$ vertical columns of $\mathbf{N}$ elements, all driven in the same phase.

(2) If we assume that no stacking is involved, then $\mathbf{N} = 1$ and we have for the test ratio for $N$ couplets

$$
\begin{aligned}
T_2 &= \frac{1}{2N} + \frac{3}{2N^2}\sum_{K=1}^{N-1}(N-K)\cdot Q(2\pi KA,\,0) \\
&= \frac{1}{2N} + \frac{3}{2N^2}\sum_{K=1}^{N-1}(N-K)\cdot\left[\frac{\sin 2\pi KA}{2\pi KA} + \frac{\cos 2\pi KA}{(2\pi KA)^2} - \frac{\sin 2\pi KA}{(2\pi KA)^3}\right]. \qquad (27)
\end{aligned}
$$

This equation was used in the calculation of the data given in Figs. 6, 7, and 8.

(3) If we wish to apply equation (25) to the case of a single array of $N$ linear oscillators driven in phase we have $n = \mathbf{N} = 1$ and $B = 0$, so

$$
T_3 = \frac{1}{N} + \frac{3}{N^2}\sum_{K=1}^{N-1}(N-K)\cdot Q(2\pi KA,\,0), \qquad (28)
$$

which differs from equation (27) by a factor of two. This indicates that an array of $N$ equiphased linear couplets gives twice the field in the preferred direction as received from $N$ equiphased linear elements radiating the same power.

(4) Applying equation (25) to the extremely simple case of one couplet, $n = 2$, $a = \frac{1}{4}$, $b = -\frac{1}{4}$ and $N = \mathbf{N} = 1$ and

$$
T_4 = \tfrac{1}{2}. \qquad (29)
$$

(5) We may calculate the test ratio for a single stack of linear couplets (earth effects not considered) by placing $N = 1$, $n = 2$, $a = \frac{1}{4}$, $b = -\frac{1}{4}$, and $\mathbf{B} = 0$ and get

$$
\begin{aligned}
T_5 &= \frac{1}{2\mathbf{N}} + \frac{3}{2\mathbf{N}^2}\sum_{\mathbf{K}=1}^{\mathbf{N}-1}(\mathbf{N}-\mathbf{K})\cdot Q(0,\,2\pi\mathbf{KA}) \\
&= \frac{1}{2\mathbf{N}} - \frac{3}{\mathbf{N}^2}\sum_{\mathbf{K}=1}^{\mathbf{N}-1}(\mathbf{N}-\mathbf{K})\left[\frac{\cos(2\pi\mathbf{KA})}{(2\pi\mathbf{KA})^2} - \frac{\sin(2\pi\mathbf{KA})}{(2\pi\mathbf{KA})^3}\right]. \qquad (30)
\end{aligned}
$$

This equation was used in calculating the data given in Figs. 18 and 19.

(6) The test ratio for the case of the rectangular array of $nN$ elements discussed in connection with Fig. 16 may be calculated by placing $N = 1$, $a = \frac{1}{4}$, $b = -\frac{1}{4}$ and $B = 0$. In which case

$$T_6 = \frac{1}{nN} + \frac{3}{nN^2}\sum_{K=1}^{N-1}(N-K)\cdot Q(2\pi KA, 0) + \frac{3}{Nn^2}\sum_{k=1}^{n-1}(n-k)\cdot\cos\frac{(k\pi)}{2}\cdot Q\left(\frac{k\pi}{2}, 0\right) + \frac{6}{n^2N^2}\sum_{K=1}^{N-1}\sum_{k=1}^{n-1}(n-k)(N-K)\cdot\cos\left(\frac{k\pi}{2}\right)\cdot Q\left(2\pi\sqrt{\frac{k^2}{16}+K^2A^2}, 0\right). \quad (31)$$

## Areas of Directional Diagrams

In general, the areas of directional diagrams may be calculated from their equations by the usual integration methods. The special case of $N$ couplets in horizontal array, such as used rather generally in practice and shown in Fig. 5 above, is of sufficient importance to be given here. The area of the diagram in the $(XY)$ plane is

$$S = \frac{1}{N^2}\left[\frac{N}{2} + \sum_{K=1}^{N-1}(N-K)\cdot J_0(2\pi KA)\cdot\cos 2\pi KB\right]. \quad (32)$$

This equation was used in calculating the data given in Fig. 5.

The area of diagrams in the horizontal plane due to a single array of $N$ oscillators is given by the equation:

$$S = \frac{2}{N^2}\left[\frac{N}{2} + \sum_{K=1}^{N-1}(N-K)\cdot J_0(2\pi KA)\cdot\cos 2\pi KB\right].* \quad (33)$$

This differs from equation (32) by a factor of two and indicates that regardless of whether the gain is reckoned by an integration over a unit sphere or in terms of the area of the horizontal diagram the effect of the reflector is to double the radiated field in the preferred direction.

Placing $N = 1$ in equation (32)

$$S = \tfrac{1}{2}. \quad (34)$$

This is analogous to equation (29) above.

* R. M. Foster, "Directive diagrams of antenna arrays," *Bell Sys. Tech. Jour.*, **5**, 307; 1926.

## Arrays of Arrays

Each element of a generalized linear array, such as shown in Fig. 21, may be replaced by a generalized array, thereby producing an array of arrays.[9] It may be shown that the resultant is given by an array factor, representing the characteristics of individual arrays, times other factors representing the relative position of the individual arrays in the array of arrays. A derivation analogous to that beginning on page 22 results in the equation

$$R = r \cdot \frac{\sin n'\pi(a' \sin \phi + b')}{n' \sin \pi(a' \sin \phi + b')} \cdot \frac{\sin N'\pi(A' \sin \phi + B')}{N' \sin \pi(A' \sin \phi + B')} \cdot \frac{\sin \mathbf{N}'\pi(\mathbf{A}' \sin \phi + \mathbf{B}')}{\mathbf{N}' \sin \pi(\mathbf{A}' \sin \phi + \mathbf{B}')}, \quad (35)$$

where $a'\lambda$, $A'\lambda$ and $\mathbf{A}'\lambda$ are the coordinate spacings between arrays and $b'T$, $B'T$, and $\mathbf{B}'T$ are the corresponding phase intervals, and $r$ represents the characteristics of one of the individual arrays. If each array is of the type shown in Fig. 5, $r$ is given by equation (14) above. Placing $n' = \mathbf{N}' = 1$ and $N' = 2$ also $n = 2$ and $B = 0$, the above equation reduces to

$$R = \frac{\sin N'\pi(A' \sin \phi + B')}{N' \sin \pi(A' \sin \phi + B')} \cdot \frac{\sin N\pi(A \sin \phi)}{N \sin \pi(A \sin \phi)} \cos \frac{\pi}{4}(1 - \cos \phi), \quad (36)$$

which is that made use of in calculating the diagrams in Figs. 14 and 15.

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